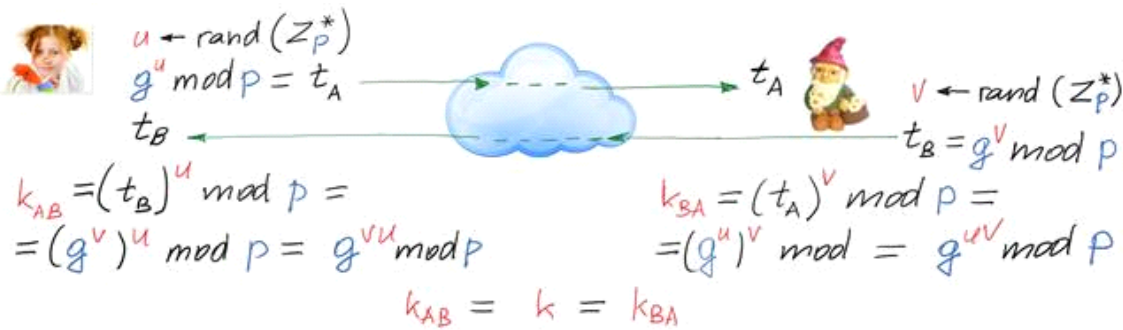


Using the same secret key Alice can encrypt a large message using block cipher for example AES-128, 192, 256.

Diffie-Hellman Key Agreement Protocol (DH KAP)

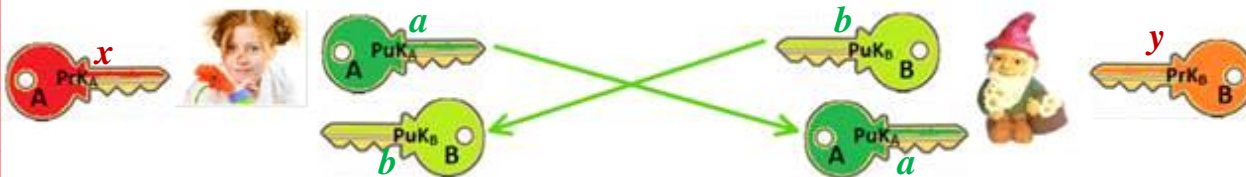
Public Parameters $PP=(p,g)$



AKAP: EIG-Enc + EIG-Sig = AKAP

Hybrid enc-dec method.

- Parties must agree on common symmetric secret key k .
for symmetric block cipher, e.g. AES-128, 192, 256.



$$A: PrK_A = x; PuK_A = a = g^x \text{ mod } p.$$

$$PuK_B = b.$$

$$B: PrK_B = y; PuK_B = b.$$

$$PuK_A = a.$$

$$1) k \leftarrow \text{rand}_i(2^{128})$$

$$i_k \leftarrow \text{rand}_i(2^{128})$$

1) $k \leftarrow \text{randi}(2^{128})$
 $i_k \leftarrow \text{randi}(2^{128})$

$\text{Enc}(\text{PuK}_B = b, i_k, k) = c = (E, D)$

2) M - large file to be encrypted

$E_k(M) = \text{AES}_k(M) = G$

3) Signs ciphertext c

3.1) $h = H(c)$

3.2) $\text{Sign}(\text{PrK}_A = x, h) = \tilde{G} = (r, s)$

c, G
 \tilde{G}, PuK_A
 Cert_A

1.1. Verify if PuK_A and Cert_A are valid?

1.2. Verify if \tilde{G} on $h = H(c)$ is valid?

$h' = H(c)$

$\text{Ver}(\text{PuK}_A, \tilde{G}, h') = \text{True}$

2. $\text{Dec}(\text{PrK}_B, c) = k$

3. $D_k(G) = \text{AES}_k(G) = M.$

A was using so called encrypt-and-sign (E-&-S) paradigm.

(E-&-S) paradigm is recommended to prevent so called Chosen Ciphertext Attacks - CCA: it is most strong attack but most complex in realization.

```
>> p=int64(268435019)
p = 268435019
>> g=2;
```

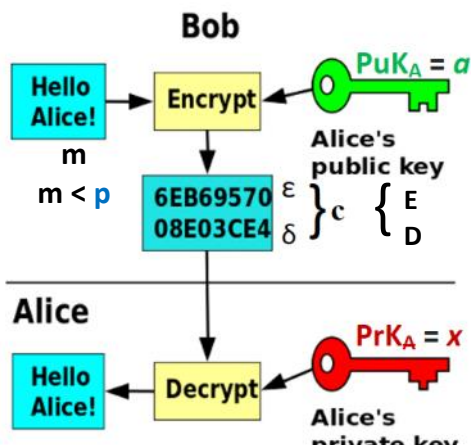
```
>> x=int64(randi(2^28-1))
x = 189519825
>> a=mod_exp(g,x,p)
a = 80162083
>> k=int64(randi(2^28))
k = 147364121
```

```
>> y=int64(randi(2^28-1))
y = 162527967
>> b=mod_exp(g,y,p)
b = 76401416
```

Asymmetric Encryption - Decryption

$c = \text{Enc}(\text{PuK}_A, m)$

$m = \text{Dec}(\text{PrK}_A, c)$

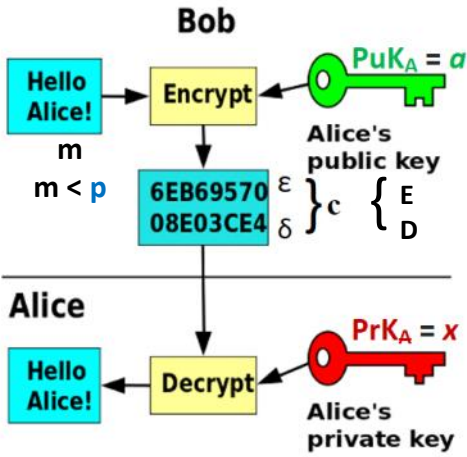


Alice is encrypting key k using ElGamal encryption

$\text{Enc}(\text{PuK}_B, i_k, k) = c = (E, D)$

$E = k \cdot b^{i_k} \text{ mod } p$; $D = g^{i_k} \text{ mod } p.$

```
>> b_ik=mod_exp(b,ik,p)
b_ik = 262733343
>> E=mod(k*b_ik,p)
E = 75434686
>> D=mod_exp(g,ik,p)
D = 2879528
```



$$enc(PuK_B, k, r) = c = (E, D)$$

$$E = k \cdot b^{ik} \text{ mod } p; D = g^{ik} \text{ mod } p.$$

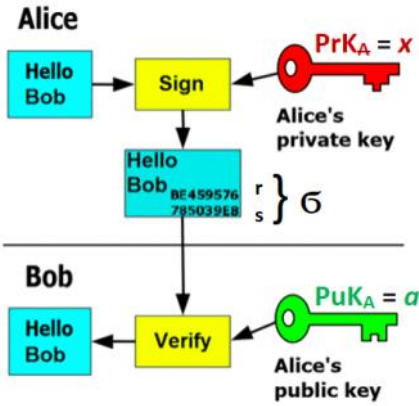
```
>> b_ik=mod_exp(b,ik,p)
b_ik = 262733343
>> E=mod(k*b_ik,p)
E = 75434686
>> D=mod_exp(g,ik,p)
D = 2879528
```

Signature creation for message $M > p$.

$$h = sha('E, D')$$

$$j = randi(2^{28})$$

$$Sign(PrK_A, j, h) = \sigma = (r, s)$$



1. Compute decimal h-value $h = H('E, D')$; $h < p$.
 2. Generate $j = \text{int64}(\text{randi}(p-1)) \% \text{ such that } \text{gcd}(j, p-1) = 1$.
 3. Compute $j^{-1} \text{ mod } (p-1)$. You can use the function
 $\gg j_m1 = \text{mulinv}(j, p-1)$;
 4. Compute $r = g^j \text{ mod } p$.
 5. Compute $s = (h - xr)j^{-1} \text{ mod } (p-1)$.
 6. Signature on h-value h is $\sigma = (r, s)$.
- $Sign(x, h) = \sigma = (r, s)$.

```
>> h=hd28('75434686,2879528')
h = 185959935
>> j=int64(randi(p-1))
j = 91638421
>> gcd(j,p-1)
ans = 1
>> j_m1=mulinv(j, p-1)
j_m1 = 123035687
>> mod(j*j_m1,p-1)
ans = 1
```

```
>> r=mod_exp(g,j,p)
r = 190075158
>> xr=mod(x*r,p-1)
xr = 47387312
>> hmxr=mod(h-xr,p-1)
hmxr = 138572623
>> s=mod(hmxr*j_m1,p-1)
s = 163470271
```

A: $c = (E, D), G, \sigma = (r, s)$ → B:

$PuK_A, Cert_A$

1. computes $h = H('E, D')$

```
>> h=hd28('75434686,2879528')
h = 185959935
```

2. Verifies signature σ on h

2. Signature Verification

A signature $\sigma=(r,s)$ on message M is verified using Public Parameters $PP=(p, g)$ and $PuK_A=a$.

1. Bob computes $h=H(M)$.

2. Bob verifies if $1 < r < p-1$ and $1 < s < p-1$.

3. Bob calculates $V1=g^h \bmod p$ and $V2=a^r r^s \bmod p$, and verifies if $V1=V2$.

The verifier Bob **accepts** a signature if all **conditions** are satisfied during the signature creation and **rejects** it otherwise.

```
>> V1=mod_exp(g,h,p)
V1 = 7325643
>> a_r=mod_exp(a,r,p)
a_r = 32677501
>> r_s=mod_exp(r,s,p)
r_s = 68068424
>> V2=mod(a_r*r_s,p)
V2 = 7325643
```

Let message m needs to be encrypted, then it must be encoded in decimal number m : $1 < m < p$.

E.g. $m = 111222$. Then $m \bmod p = m$.

Since key k is encrypted then $1 < k < p$.

3. Decrypts ciphertext $c = (E, D)$ to find k .
using his PrK_B .

$$3.1. D^{-x \bmod (p-1)} \bmod p$$

$$3.2. E \cdot D^{-x \bmod p} = k. \quad k = 147364121$$